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PROJECT APOLLO

A TWO-BODY ANALYSIS OF ΔV REQUIREMENTS NECESSARY FOR THE ABORT FROM A TRANSLUNAR MISSION

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SUMMARY

It is shown that abort is possible, neglecting non-two-body forces, from any radial distance from the earth to the lunar sphere of influence, from a given nominal translunar trajectory. If the return perigee velocity is about 36,000 feet per second, abort is possible if a minimum of 3000 feet per second ΔV capability is on board the spacecraft. Parabolic and hyperbolic abort returns require ΔV capabilities in excess of 7500 feet per second and perigee velocities greater than 36,451 feet per second.

INTRODUCTION

This paper will present the results of a study made by the Theoretical Mechanics Branch to investigate the fuel requirements, return times, and reentry velocities involved in aborting an Apollo mission during the translunar phase up to approximately the sphere of influence of the moon. The analysis presented ignores the presence in space of all bodies except the earth, which is assumed to be a homogenous sphere. The Apollo spacecraft is assumed to be a body of negligible mass. The analysis is also restricted to a plane. The author recognizes that the results are valid only for one nominal translunar trajectory.

ANALYSIS

The quantities assumed to be specific at the initial time, that is, at abort are V_E, the velocity of the spacecraft with respect to the earth; γ_E, the angle between V_E and the local horizontal; and r_i, the radial distance of the spacecraft from the center of the earth. Also specified are the final conditions for the return trajectory, V_p, the velocity at perigee and h_p, the perigee altitude. See figure 1.

Given these final conditions, the semi-major axis and the eccentricity of the return trajectory are immediately found to be,

$$(1) \qquad a = \left(\frac{V_p^2}{\mu} - \frac{2}{h_p + R}\right)^{-1}$$

$$(2) \qquad e = 1 + \frac{r_p}{a} \quad , \quad r_p = h_p + R$$

where, μ = universal gravitational constant times the mass of the earth.

R = radius of the earth.

The initial velocity and flight path angle of the transfer orbit are found from,

$$(3) \quad V_i = \sqrt{\mu\left(\frac{2}{r_i} - \frac{1}{a}\right)}$$

$$(4) \quad \gamma_i = \pm \cos^{-1}\left(\frac{r_P V_P}{r_i V_i}\right)$$

The ΔV required to initiate the abort is found from

$$(5) \quad \Delta V = \sqrt{V_i^2 + V_E^2 - 2 V_i V_E \cos(\gamma_E + |\gamma_i|)}$$

The time from abort to perigee may be calculated from

$$(6) \quad T_P = \frac{P}{2\pi}(E - e\sin E) \qquad \text{if } a > 0,\ \gamma_i < 0$$

or,

$$(7) \quad T_P = P - \frac{P}{2\pi}(E - e\sin E) \qquad \text{if } a > 0,\ \gamma_i > 0$$

or,

$$(8) \quad T_p = \frac{P}{2\pi}\left[e \tan H - \ln \tan\left(\frac{\pi}{4} + \frac{H}{2}\right)\right].$$

if $a < 0, \ \gamma_i < 0$

where, $E = \cos^{-1}\left[\frac{1}{e}\left(1 - \frac{r_i}{a}\right)\right]$

$H = \cos^{-1}\left[\frac{e}{1 + r_i/a}\right]$

and $P = 2\pi\sqrt{\frac{|a|^3}{\mu}}$.

The return trajectory is elliptical if $a > 0$ and is hyperbolic if $a < 0$. The sign of the semimajor axis depends upon the specified quantities V_p and h_p as is evident from equation (1). The sign of γ_i however, may be chosen. If γ_i is chosen to be positive, sketch 1a, then the return is made through apogee, requiring in general moderate ΔV for abort but large return times. If γ_i is chosen to be negative, sketch 1b, the return is direct to perigee, requiring large ΔV but small return times. For V_p and h_p such that $a < 0$, the sign of γ_i can only be negative, since a positive γ_i would probably result in escape. Two special cases, return from apogee and parabolic return, should now be discussed.

The return from apogee case is of interest since by specifying $\gamma_i = 0$ in (4) and solving for V_p, the minimum possible value of the perigee velocity is attained for a given radial distance from the earth as shown in figure 1a. There is only one possible combination of ΔV required for abort and return time, T_p, in this case.

The parabolic return is of interest since it represents the boundary between elliptical and hyperbolic returns. In the elliptical case two combinations of ΔV and T_p are possible for each specified value of perigee velocity and initial radius, whereas for hyperbolic returns only one ΔV and T_p combination is possible. The parabolic return choice would be very difficult to attain in practice since e must be specified as unity and a as infinity. The time from abort to perigee is,

$$(9) \quad T_p = \sqrt{\frac{2 r_p^3}{\mu}} \left(\tan \frac{\eta}{2} + \frac{1}{3} \tan^3 \frac{\eta}{2} \right)$$

where, $$\eta = \cos^{-1}\left(\frac{2 r_p}{r_i} - 1\right)$$

The perigee velocity cannot be specified for the parabolic return, but must be calculated from,

$$(10) \quad V_p = \sqrt{\frac{2\mu}{r_p}}$$

If, $V_p < \sqrt{\frac{2\mu}{r_p}}$ the return is elliptic.

If, $V_p > \sqrt{\frac{2\mu}{r_p}}$ the return is hyperbolic.

Scope of Calculations

Equations (5) through ($\overset{7}{8}$) were used to calculate return times and ΔV requirements for abort from a number of positions on a typical trans-lunar trajectory, tabulated in Table I. This reference trajectory was calculated from a numerical solution of the restricted three-body system (Earth-Moon-Spacecraft). The distances from the center of the earth at which aborts are initiated varied from 8707 statute miles to 215,266 statute miles, or to about 10,000 miles inside the lunar sphere of influence. The abort perigee altitude, h_p, was taken to be 50 statute miles and the perigee velocity, V_p, was varied from 32,000 to 40,000 feet per second.

Discussion of Results

Variation of the time from abort to perigee with the ΔV or velocity increment required to initiate the abort is shown in figure 2. It is evident that for radial distances up to 0.7 of the average lunar distance abort may occur with perigee velocities of less than 36,000 feet per second, and furthermore this return may be direct $(-\gamma_L)$ or through apogee $(+\gamma_L)$. For these distances return may also occur from apogee or by parabolic and hyperbolic trajectories. As expected, aborts through apogee require less ΔV but greater return times than direct aborts.

Abort is possible from any radial position provided that the ΔV required for the abort is available on board and the return perigee velocity is not so great that problems with the heat shield during reentry to the earth's atmosphere are encountered. If a ΔV capability of at least 3,000 feet per second is available, abort is possible up to the lunar sphere of influence, $r_L = 0.85$ with a return perigee velocity of about 36,000 feet per second. With this ΔV capability, the maximum return time is about 82 hours at a radial position of 0.7 and about 14 hours at a radial distance of about 0.04. It should be noted from figure 2 that abort returns on parabolic or hyperbolic trajectories from any radial distance require ΔV in excess of 7500 feet per second and have perigee velocities in excess of 36,451 feet per second.

Radial distances larger than about 0.85 of the lunar distance are beyond the scope of the two-body analysis presented in this paper. Abort from these distances may only be studied by a more sophisticated analysis that includes the gravitational effects of the moon and sun. Since the results are valid only for a particular nominal translunar trajectory, a more sophisticated treatment should also include the variation of ΔV to abort, time to return to perigee, and perigee velocity with variations in the nominal trajectory.

Concluding Remarks

In general, abort is possible from all radial distances. From any radial distance short of the lunar sphere of influence abort is possible with perigee velocity of about 36,000 fps if a minimum of 3,000-feet-per-second ΔV is available on board the spacecraft. Parabolic and hyperbolic returns require ΔV capabilities of 7500 feet per second or greater and have perigee velocities greater than 36,451 feet per second.

Time from Insertion HRS	Radius		Velocity, V_E, fps	Flight Path Angle, γ_E, deg
	Miles	Fraction of Lunar Distance From Earth		
0.489	8,707	.04	24,358	133.62
1.011	14,653	.06	18,571	122.53
2.300	26,732	.11	13,435	114.05
6.211	53,523	.22	8,981	107.72
16.700	102,013	.43	5,774	104.35
38.033	166,143	.70	3,650	103.48
62.211	215,266	.90	2,630	95.49

TABLE I: Reference Trajectory From Which Aborts Were Initiated.

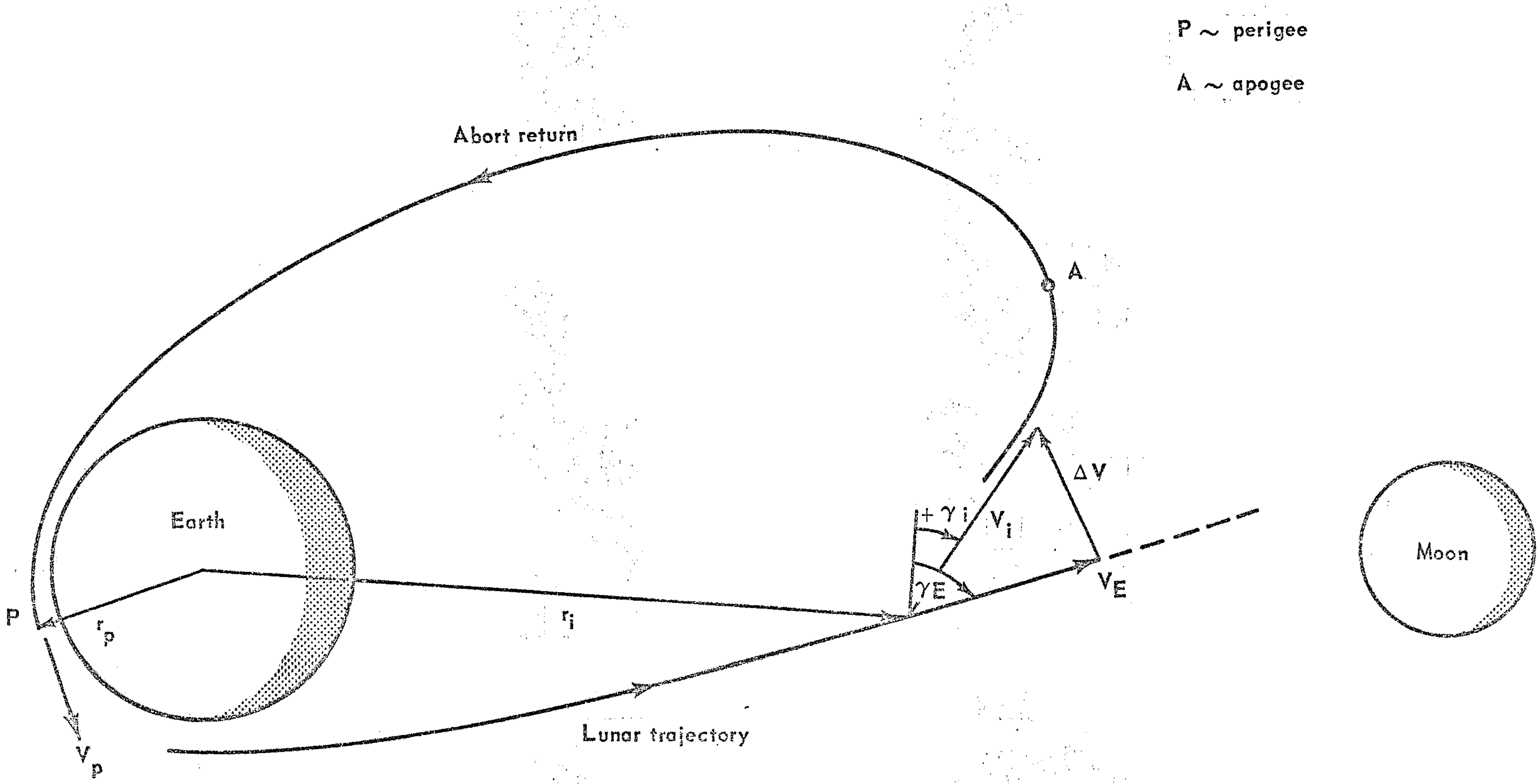

Figure 1a Abort from lunar trajectory with return through apogee

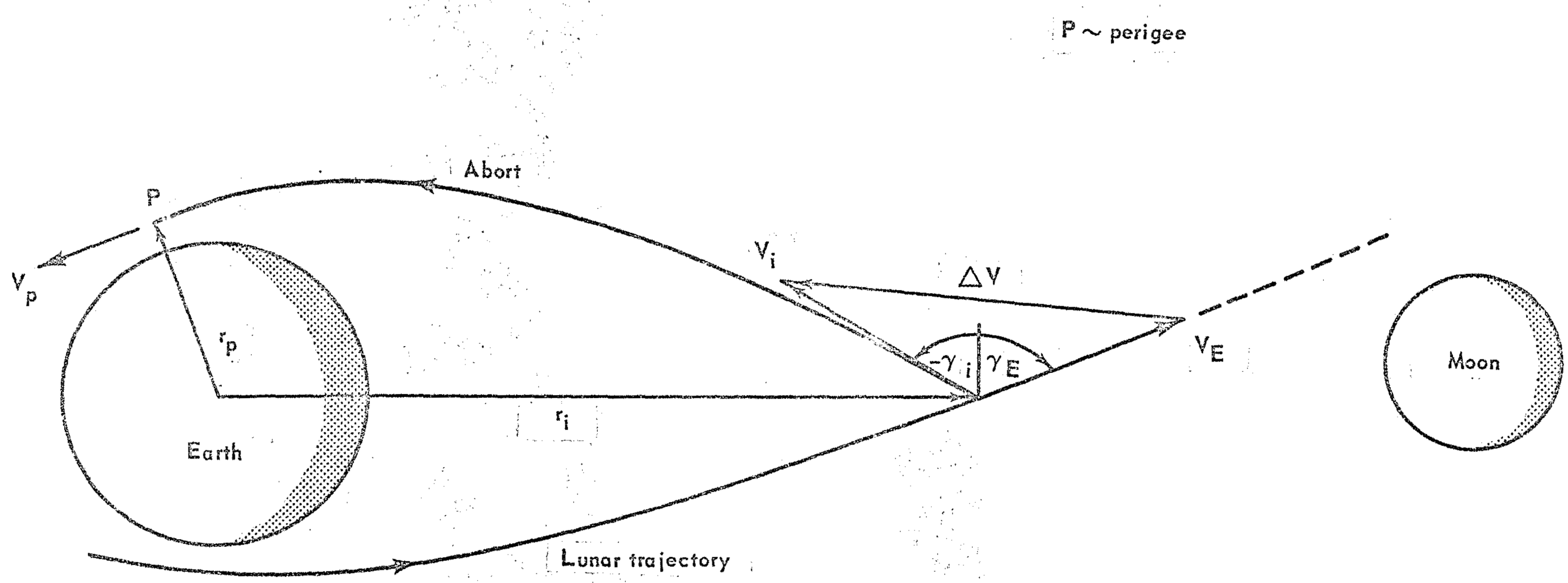

Figure 1b Abort from lunar trajectory with a direct return

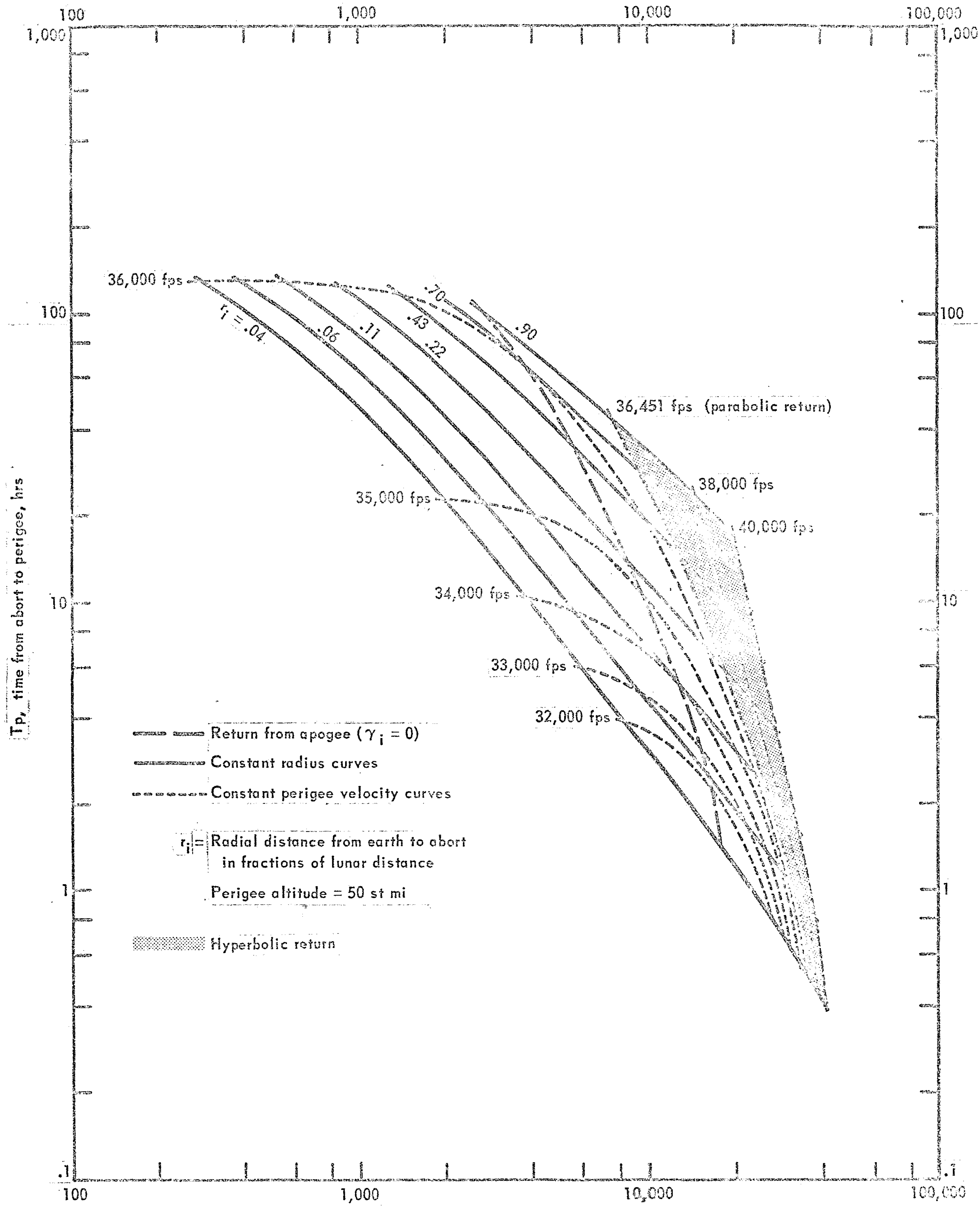

Figure 2 Velocity increment to initiate abort, fps